(Not necessarily correct, please find any mistakes and comment them on the side bar)

**Section A**

am bad at maths pls halp

**1a**

Take and . We know that (since )and . Hence, by the sandwich theorem, as .

**1b**

We compare to .

…

….

By comparison a\_n diverges.

**Section B**

**3a**

i) {2, 5, 8}

ii) {3, 6, 9}

iii) {1, 4, 7}

iv) {1, 3, 4, 6, 7, 9}

**3b**

i) bijective

ii) Not onto ( ¬) and not one-to-one ()

iii) one-to-one

iv) bijection,

**3c**

If , it means that a bijection , where exists. By examining the domain, we can see that and . However, for all in , we get in . Thus, for all in , the same element exists in Thus all elements are in Thus, Similarly, Thus

**3d**

If two cases are possible:

If . However, a bijection guarantees that the domain and the image have the same cardinality. (Notes) Thus,

**3e**

*Since word doesn’t have \bowtie, I will use instead.*

* Reflexive:

. True since true.

* Symmetric:

True since:

* Transitive

Assume:

Then:

By adding (1) and (2), by parts and algebra, we get:

. Thus, transitive.

**Graphs Questions:**

**Q4**

1. MST - T is a spanning tree for G and no other spanning tree has a lower total weight than T
   1. A, B, H, G, C, F, E, D



* 1. Yes, the arc weights, in order, are 1,2,3,3,4,4,5,5,6,8,10,10. There are 8 nodes in G, and so 7 arcs are required for a MST. The 7 smallest weighted arcs have weights 1,2,3,3,4,4,5, which are the arc weights which make up the MST above. The other ac of weight 5 cannot be used as it would create a cycle ABGHA, so replacing any arc with one not in the MST would either create a cycle, so it is not a tree, or increase the overall weight, so it is not minimal, so this MST is unique.

ci.)

If you repeatedly remove the largest arc in a cycle you will eventually end up with no more arcs to remove since every cycle has a finite number of arcs

cii.)

**Base case V**

**To show**: G0 connected

**Proof**: G0 = G, and from the question we know that G is a ”connected weighted graph” so G0 isconnected since G0 = G.

**Inductive step**

Ind hyp: Gk connected

To show: G(k+1) is connected

**Proof**

To go from G(k) to G(k+1) we remove an arc that is part of a cycle. This arc joins nodes n1 and n2. Since Gk is connected, G(k+1) will only be connected if n1 and n2 are still connected to the other nodes. This is true because the arc was removed from a cycle, so there exists another path from n1 and n2 to the other nodes.

**Deduction**

Since the algorithm terminates when there are no cycles left (Q4ci) and Gi is always connected according to the above proof, then when the graph terminates we are left with a connected graph with no cycles which is by definition a tree.